Criteria for the Bifurcation to Wavy Taylor Vortex Flow with a Small Aspect Ratio in a Numerical Analysis

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Abstract: This study presents some criteria for the distinction between the Taylor vortex flow and the wavy Taylor vortex by using a three-dimensional numerical calculation. We prepare four kinds of the criteria. The first criterion is to check vector diagram of velocity. The diagram in the wavy Taylor vortex flow showed that the vector lines near by the boundary surfaces between the cells vibrated in the axial direction. Second and third approaches are to check the variance of the spatially averaged energy. The energy averaged by the whole volume of the flow field in the wavy Taylor vortex vibrated with the time excepting in the neighborhood of the critical parameter. Individual energy averaged by the volume of each cell in the wavy Taylor vortex flow could show a little variance of the energy in the neighborhood of the critical parameter. As an additional approach, attractor in the topological space was drawn. Consequently the numerical results for the critical Reynolds number were estimated and compared with the experimental results.

Keywords: Numerical Analysis, Fluid Dynamics, Nonlinear Dynamics, Bifurcation, Taylor Vortex Flow

1. INTRODUCTION

We present a numerical analysis for the bifurcation to the wavy Taylor vortex flow from the Taylor vortex flow with a small aspect ratio. This subject has first presented with by T. Mullin and T. B. Benjamin [1]. In our previous report [2], the formation process to the Taylor vortex flow was shown by the vector diagram and the energy averaged by the whole volume of the flow field was shown. And the critical Reynolds number to the wavy Taylor vortex flow was revealed. However the quantitative agreement between the numerical results and the experimental results was not quite enough. That would be why that the numbers of the grid points were not the most suitable. Above all the criteria for the distinction for the Taylor vortex flow and wavy Taylor vortex flow have not been defined yet. Our purpose is to define standard criteria for the distinction between the Taylor vortex flow and the wavy Taylor vortex flow. In this 3-dimensional numerical analysis, the more grid points were adopted. And we adopt four kinds of the approaches for the criteria. The first criterion is to use a vector diagram of the velocity. It is easy before everything to check the vector lines of the flow by naked eyes. And the second criterion is to check the variance of the spatially averaged energy. The energy averaged by the whole volume of the flow field in the wavy Taylor vortex showed the vibration with the time. The third one is to check the individual energy averaged by the volume of each cell. This reveals a little variance of the energy in the wavy Taylor vortex flow. As an additional approach, attractor in the topological space is used [3]. Finally the numerical results of the critical Reynolds number are estimated and compared with the experimental results.

2. BASIC EQUATION

Governing equations are Navier-Stokes equation and continuity equation in the cylindrical coordinate system (r, θ, z) as shown in the followings but in the vectorial notation.

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \mathbf{\nabla} \times \mathbf{u}
\]

\[
\mathbf{\nabla} \cdot \mathbf{u} = 0.
\]

Where \( \mathbf{u} = (u, v, w) \). A kind of stream functions \( \Psi \) that is similar to the Stokes’ stream function is adopted for flow visualization. Radius component \( u \) and axial component \( w \) are expressed as a function \( \Psi \) as follows

\[
\mathbf{u} = \frac{1}{r} \frac{\partial \Psi}{\partial z}, \quad w = \frac{1}{r} \frac{\partial \Psi}{\partial r}.
\]

The spatially averaged energy is defined as follows

\[
E = \frac{1}{V} \int_V \frac{1}{2} |\mathbf{u}|^2 dV.
\]
Here $V$ is the total volume occupied by flow or the volume occupied by each cell flow occasionally.

3. NUMERICAL METHODS AND NUMERICAL EXPERIMENT

The MAC method was used as a basic solution procedure. The time integration was explicit method and spatial differentiation was the QUICK method for the convection terms and the second-order central difference method for other terms. A hybrid method of SOR and ILUCGS was used to solve the Poisson equation. In the previous work, the numbers of the grid points of the radial, the azimuthal and the axial direction were used 21, 20, 84 at the aspect ratio four respectively. However the numerical results were not necessarily good agreement with the experimental results. So in this work, we selected the numbers of staggered grid points 41 in the radial direction, 74 in the azimuthal direction. And the grid point in the axial direction was defined as an expression, ($\varpi$ = 41)-1. So the number of the grid point at aspect ratio four is 163. Time interval was used 0.0025. It has two times precision as previous analysis.

In the numerical experiment, the Reynolds number at first increased up to the 1800 to make the wavy Taylor vortex flow. Then the Reynolds number is decreased to an established value gradually step by 90. The judgment whether the mode is the Taylor vortex flow or the wavy Taylor vortex flow was checked by the vector lines, the energy of the whole flow field and/or the energy of the each cell. These processes were repeated until the definition of the critical Reynolds number. Finally the numerical results were compared with the experimental results.

![Figure 1. Development of the normal 2-cell mode](image1)

(a) Taylor vortex flow $\varpi=1.0$, Re=675  
(b) Wavy Taylor vortex flow $\varpi=1.0$, Re=855

![Figure 2. Energy in the radial direction averaged by whole volume](image2)

(a) Taylor vortex flow $\varpi=1.0$, Re=675  
(b) Wavy Taylor vortex flow $\varpi=1.0$, Re=855
4. RESULTS AND CONSIDERATION

Figure 1 shows the vector diagrams of the Taylor vortex flow (TVF) and the wavy Taylor vortex flow (WTVF) at $\pi=1.0$. The boundary line is stable in TVF but is unstable up and down in WTVF. Most of the cases, the estimation whether the flow mode is TVF or WTVF is easy because the boundary curves in the velocity lines corresponding the outward flow of the WTVF are definitely oscillating. Therefore, this method would be useful to the distinction between the TVF and WYVF. However in the neighborhood of the critical Reynolds number, it is difficult to judge a little oscillation by naked eyes.

Figure 2 shows the diagrams of the spatially averaged energy that are averaged by the whole volume of the flow field. Generally, in the final stable mode, the energy of the TVF should be constant but that of the WTVF would be oscillating. Therefore this method can show distinction of two modes. However it is difficult to judge in the critical region because small changes of the energy of the each cell could be hidden by the average by the whole volume as shown Fig.2.

Figure 3 shows the diagrams of the each energy averaged by the each cell volume. These diagrams could show the energy of the upper cell and the bottom cell in the 2-cell mode. Both energies, both lines are coincident and so you could see one line, are stable in TVF in Fig.3 (a) and the energies of the upper cell and the bottom cell

(a) Taylor vortex flow $\pi=1.0$, Re=675

(b) Wavy Taylor vortex flow $\pi=1.0$, Re=855

The delay time was defined from auto correlation function.
are oscillating in WTVF in Fig.3 (b). This criterion can detect the small difference of the energy of the cell and distinct the difference between TVF and WTVF.

Figure 4 shows the topological attractors of the energy. The delay time $\Delta t$ was determined as a optimum from a autocorrelation function. The attractor of the TVF draws the limit cycle and that of the WTVF draws the torus or the resemble torus. This criterion would be useful for the distinction of the modes. However, the criterion should be treated carefully how to select the delay time and how to check the attractor. Because the torus depends on the delay time and the change of the energy, therefore it might have some kinds of shape according to them.

Figure 5 show the comparison between the numerical results and the experimental results. The white marks show the critical Reynolds numbers to the WTVF from the TVF in the experiment. And the black marks are those in the numerical analysis. The both color circles show the normal 2-cell mode, the square marks show the normal 4-cell mode. These quantitative and qualitative results show the better agreement than our previous results.

5. CONCLUSION
We present the criteria for the distinction between the Taylor vortex flow and the wavy Taylor vortex flow by using the three-dimensional numerical calculation. Four kinds of the criteria are of the vector diagram, the averaged energies by the volume of the whole flow field, the averaged energies by the volume of each cell and the attractor in the topological space. It was confirmed that all criteria could be useful for the distinction. Each criterion has a characteristic and it should be adapted to the occasion. In any case the combination of the criteria should be used for the distinction of the modes. Finally, the comparison between the numerical and experimental results was better agreement.

REFERENCES